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Nonholonomic mobile robots' trajectory tracking model predictive control: a survey

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SUMMARY
Model predictive control (MPC) theory has gained attention with the recent increase in the processing power of computers that are now able to perform the needed calculations for this technique. This kind of control algorithms can achieve better results in trajectory tracking control of mobile robots than classical control approaches. In this paper, we present a review of recent developments in trajectory tracking control of mobile robot systems using model predictive control theory, especially when nonholonomicity is present. Furthermore, we point out the growth of the related research starting with the boom of mobile robotics in the 90s and discuss reported field applications of the described control problem. The objective of this paper is to provide a unified and accessible presentation, placing the classical model, problem formulations and approaches into a proper context and to become a starting point for researchers who are initiating their endeavors in linear/nonlinear MPC applied to nonholonomic mobile robots. Finally, this work aims to present a comprehensive review of the recent breakthroughs in the field, providing links to the most interesting and successful works, including our contributions to state-of-the-art.

KEYWORDS: Model predictive control; Mobile robots; Control of robotic systems; Trajectory tracking; Navigation.

1. Introduction
Trajectory tracking problems for autonomous vehicles are usually solved by designing control laws that make the vehicles track predetermined, feasible trajectories.[3] However, this approach suffers from drawbacks mainly due to the vehicles' dynamics that exhibits complex nonlinear terms and significant uncertainties, which makes the task of computing a feasible trajectory very difficult. In some cases, especially in the presence of tracking errors, the controller attempts to make the outputs catch up with the time-parameterized desired outputs. This may lead to closed-loop performance difficulties and too large control signals.[47]

Path following problems are primarily concerned with the design of control laws that steer an object to reach and to follow a geometric path, while a secondary goal is to force the object moving along the path to satisfy some additional dynamic specifications. In our work, we address model predictive control (MPC) theory applied to mobile robots path tracking, especially in the case of nonholonomic mobile robots. Both linear and nonlinear approaches are brought into discussion as well as the main current issues being researched regarding MPC/NMPC (nonlinear model predictive control) approaches to mobile robotics. This survey does not focus on MPC theory in a global fashion rather focusing in its application to mobile robotics. Mobile robots systems are highly dynamic

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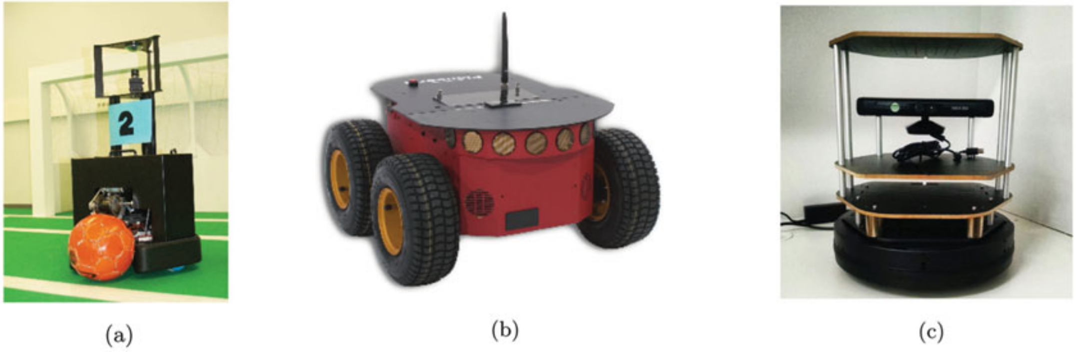


Fig. 1. Mobile robots. (a) Omnidirectional. (b) Outdoor differential-drive. (c) Indoor differential-drive.

systems that end up inserting several constraints in the control system, such as response speed, precision, mobility constraints, computational cost, computational power, maneuverability, and control stability issues.

In Section 2, we give an overview of mobile robots systems. Section 4 discusses about the MPC/NMPC approaches applied in mobile robot motion control. Section 3 enumerates the control problems currently being studied when a model predictive controller (linear or nonlinear) is applied to mobile robot for path tracking, concluding this survey in the last section.

2. Nonholonomic Mobile Robots – An Overview

A mobile robot is a highly dynamic system that relies on locomotion mechanisms to move throughout its environment within a large variety of possible ways to move. There are mobile robots that can walk, jump, run,[103] slide, skate, swim,[90] fly,[42] and roll.[75] Whereas most of these locomotion mechanisms have been inspired by their biological counterparts, there is one exception—the actively powered wheel.[91]

There are several wheel configurations that can create different wheeled mobile robots (WMRs) based on four different types of wheels.[20,58,90] The four major classes of wheels are the following: standard wheel – two degrees of freedom; castor wheel – two degrees of freedom; Swedish wheel – three degrees of freedom; and ball or spherical wheel. These four class of wheels lead to 17 different mobile robot configurations with the number of wheels varying from 2 up to 6 wheels. The most commonly used configurations in research mobile robotics are as follows:

1. Bicycle and balancing: two nonholonomic configurations with two wheels being both used as simplified models of other mobile robots with four wheels or analyzed in regard to its balance control issue.
2. Differential drive (unicycle): two nonholonomic configurations being possibly mechanically designed with three or four wheels (Fig. 1(c)).
3. Ackerman-steered wheel: a nonholonomic configuration with one or two motorized wheels in the rear and one or two steered wheels in the front. The steering has to be different for the two wheels to avoid slipping/skidding.[84]
4. Omnidirectional: an holonomic configuration with three or four Swedish wheels and omnidirectional movement (Fig. 1(a)).

2.1. Indoor versus outdoor configuration

Balance is usually not a research problem in wheeled mobile robotics because robots are almost always designed so that all of the wheels are generally in contact with ground. When more than three wheels are used, a suspension system is required to allow all wheels to maintain ground contact when the robot encounters uneven terrain (Fig. 1(b)). Therefore, wheeled robot research tends to focus on the problems of traction and stability, maneuverability, and control. This can be further analyzed regarding the environment they are. Regarding outdoor mobile robots, usually researchers need to take into account problems such as the following:

1. Higher sensor noise, e.g., in vision systems due to illumination variation;
2. Weather variation, such as fog, snow, wet floors, mud, sand, etc.;
3. Floor irregularities;
4. Localization and state estimation issues.

All above-mentioned issues affect directly the mobile robot trajectory tracking control system. Only a few works in literature uses outdoor WMRs with model predictive control that make this an open topic for researchers. An example is the work by Panathula *et al.*[80] where they present the MPC of Hilare-type robot on a slippery 3D terrain having known ground conditions while avoiding wheel slip. Their work was validated only in simulation. In real robot experiments, the works by Backman *et al.*[10,11] present a NMPC applied to a tractor–trailer system. The control system also considered the collision avoidance problem.

In 2014, Ostafew *et al.*[76] proposed a learning-based nonlinear model predictive control (LB-NMPC) algorithm for an autonomous mobile robot to reduce path-tracking errors over repeated traverses along a reference path. The LB-NMPC algorithm uses a simple *a priori* vehicle model and a adaptive disturbance model. Disturbances are modeled as a Gaussian process (GP) based on data collected during previous traversals as a function of system state, input and other relevant variables. The authors validated they work with experimental results including over 1.8 km of travel by a four-wheeled, 50 kg robot traveling through challenging terrain (including steep, uneven hills) and by a six-wheeled, 160 kg robot learning disturbances caused by nonmodeled dynamics at speeds ranging from 0.35 m/s to 1.0 m/s. Meanwhile, Lim *et al.*[58] presented a nonlinear model predictive tracking control scheme for a six-wheeled nonholonomic unmanned ground vehicle (UGV). They employed a high-level guidance control with kinematic approximation for UGV motion and a NMPC algorithm to solve trajectory planning and optimal control problems. The last work on outdoor mobile robots is an update from the work of Ostafew *et al*.[77] In contrast, although indoor mobile robots are often studied due to controllable environments, other issues can be investigated in detail.

2.2. Nonholonomic indoor mobile robots

The differential drive nonholonomic subclass of mobile robots is popular for achieving a naturally high stability. In such robots, motion in a particular direction may initially require a rotational motion. Such a robot can spin without changing its ground footprint with two-wheel differential-drive configuration where the two wheels rotate around the center point of the robot. One or two additional ground contact points may be used for stability, based on the application specificity. This configuration is especially challenging due to its maneuverability and controllability constraints.

There is generally an inverse relationship between controllability and maneuverability. For example, an omnidirectional design such as the four-caster wheels configuration requires significant processing to convert desired rotational and translational velocities to individual wheel commands. Furthermore, such omnidirectional designs often have greater degrees of freedom at the wheel. These degrees of freedom cause an accumulation of slippage, tend to reduce dead-reckoning accuracy, and increase the design complexity.

In contrast, a differential-drive mobile robot can go straight simply by rotating both driving wheels at the same direction with the same absolute velocity, even though this can be challenging considering variations between wheels, motors, and environmental differences. With a four-wheel omnidirectional robot (four Swedish wheels), the problem is even harder because all four wheels must be driven at exactly the same speed for the robot to travel in a perfectly straight line. In summary, there is no "ideal" drive configuration that simultaneously maximizes stability, maneuverability, and controllability. In this paper, we will not focus on the robot configuration, although it is more common to find research works with differential-drive mobile robots, a nonholonomic configuration that can be difficult both to maneuver and to control due to its constraints.

This paper will then focus on indoor robots specially with nonholonomic constraints. The following topics present a discussion about the current issues and research works onMPC approaches (linear and nonlinear) regarding the trajectory tracking problem of indoor mobile robots.

3. Model Predictive Control Approaches

Nonholonomic mobile robot motion control problems have been studied during the past two decades, so far, and the relevant works found are mainly concerned with obtaining feedback laws that guarantee the asymptotically stable equilibrium of the closed loop.[41] According to the Brockett theorem, a smooth, time-invariant, static-state feedback control law cannot be used to stabilize a nonholonomic system at a given configuration.[45] In the last 20 years, MPC theory has been trying to solve this control problem. MPC aims to solve optimization problems using a prediction horizon of the system's behavior with input and state constraints. A major concern in the use of a prediction horizon is to determine whether such an open loop control can guarantee system stability. It is shown that an infinite predictive horizon can guarantee stability of a system, but the choice of an infinite predictive horizon may not be feasible for a nonlinear system in practice.[23]

One of the first surveys dealing with nonholonomic control problems was written by Kolmanovsky and McClamroch.[51] In the survey, the authors mention two problems (motion planning control systems and feedback stabilization) and point out three other important issues: models of nonholonomic control systems, new control approaches for motion planning for nonholonomic systems, and stabilization of these new approaches. Nonetheless, new approaches in optimal control arise afterwards.[25,44,83] One of the approaches introduced is MPC. The MPC problem has its origin in the late 70s and has been developing considerably since then.[16] The term MPC (also called receding horizon predictive control, or RHPC) does not designate a specific controller but rather an ample range of control methods that make explicit use of a model of the process to obtain the control signal by minimizing the cost function. These design methods lead to controllers that have practically the same structure and present adequate degrees of freedom. The various MPC algorithms differ among themselves only in the model used to represent the process, the noise representation, and in the method/algorithm to minimize the cost function. The most important concepts appearing in the predictive control family are basically:

1. explicit use of a model (linear or nonlinear) to predict the process output at future time instants (prediction horizon);
2. calculation of a control sequence minimizing a cost function;
3. receding strategy, so that at each instant the horizon is displaced toward the future.

According to Findeisen and Allgöwer,[28] the MPC problem is formulated as solving, on-line, a finite horizon open-loop optimal control problem subject to system dynamics and constraints involving states and controls. Based on the model and on measurements obtained at time t, the controller predicts the future dynamic behavior of the system over a prediction horizon T_p and determines (over a control horizon $T_c \leq T_p$) the input such that a predetermined open-loop performance objective functional is optimized. If there are no disturbances and no model-plant mismatch, and if the optimization problem can be solved for infinite horizons, then one can apply the input function found at time $t = 0$ to the system for all times $t \geq 0$. However, notice that this is not possible in general. Due to disturbances and model-plant mismatch, the true system behavior is different from the predicted behavior. In order to incorporate some feedback mechanism, the obtained open-loop manipulated input function is implemented only until the next measurement becomes available. The time difference between the recalculation/measurements can vary. However, it is often assumed as fixed, i.e., the measurement takes place every δ sampling time-units. Using the new measurement at time $t + \delta$, the whole procedure (prediction and optimization) is repeated in order to find a new input function with the control and prediction horizons moving forward.

Furthermore, the optimization of the cost function has several peculiarities, regarding how it is defined. If the criterion used in the cost function is quadratic, and the model is linear, and there are no constraints on the system, then an explicit solution can be found off-line through the implementation of a simple look-up table (different gains for different operation points) or an explicit function of past inputs, outputs, and future set-point/trajectory if available. In the usual case of nonlinear constraints or when the cost function takes a less usual form, it is necessary to do a real-time on-line optimization, using a numerical method. In order to guarantee the convergence of minimization and to achieve smooth control signals, it is also customary to include in the cost function a term that penalizes the control effort. This control strategy is typically implemented using the basic structure of the model presented at Fig. 2.

The NMPC ability of enabling a robot to track a trajectory is due to the fact that the cost functions used by the controller minimizes a deviation of the predicted behavior of the robot from the desired

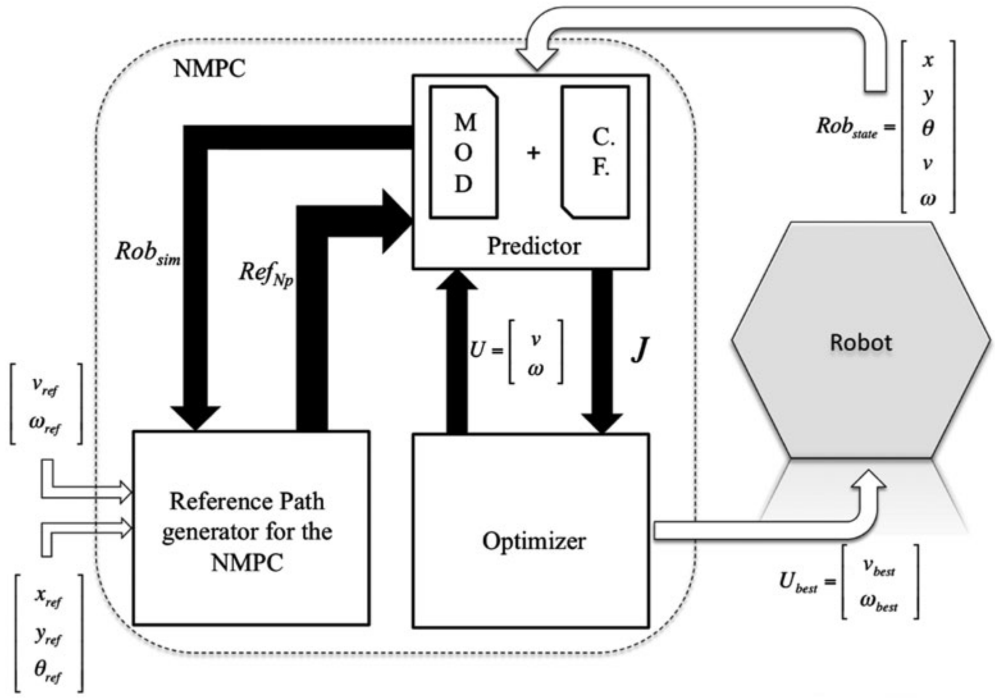


Fig. 2. Generic MPC structure, showing the control strategy generally used.

trajectory. The minimization by the optimizer and the predicted behavior are the main blocks of the MPC theory. When applied to solve the trajectory tracking control problem, the NMPC gains a new sub-part being thus divided into three sub-blocks:

- *Path Generator* – This sub-block receives the global trajectory calculated by a trajectory generator and creates a reference signal to be followed by the controller during the prediction horizon.
- *Optimizer* – This sub-block uses an on-line numeric minimization method to optimize the cost function and generate the signals of optimal control; in MPC algorithms applied to mobile robotics, it usually uses the gradient descent algorithm.
- *Predictor* – The predictor computes the predicted state evolution of the robot itself; it uses a simplified robot model to emulate the robot evolution and calculate the final value of the cost function.

Figure 2 illustrates the structure that NMPC uses, where $U_{best}(k) = U(k|k) = [\, v_{best}(k) \;\; w_{best}(k)\,]^{\mathrm{T}}$ is the output control signal, $U = \hat{U}(k+i|k)$ with $i = 0 \ldots N_c - 1$ is the output control signal from the Optimizer sent to the Predictor, and $J = \hat{J}(k+i|k)$ with $i = 1 \ldots N_p$ is the response of the predictor block to each $\hat{U}(k+i|k)$. At a given instant k, the robot sends its pose $Rob_{state}(k) = [\, x(k) \;\; y(k) \;\; \theta(k) \;\; v(k) \;\; \omega(k)\,]^{\mathrm{T}}$ to NMPC to be used by the Predictor sub-block. The NMPC also receives the reference pose sent by the trajectory generator in the world frame $P_{\mathrm{t}}(k) = [\, x_{ref}(k) \;\; y_{ref}(k) \;\; \theta_{ref}(k)\,]^{\mathrm{T}}$ and the reference velocity $V_{\mathrm{t}}(k) = [\, V_{ref}(k) \;\; \omega_{ref}(k)\,]^{\mathrm{T}}$.

After receiving the reference pose and velocity, the controller Reference Path Generator creates another path to follow within the prediction horizon (Ref_{Np}) and sends it to the Predictor sub-block. Meanwhile, this sub-block also receives the state of the robot (Rob_{state}). Also, the Optimizer sub-block provides the control input $\hat{U}(k+i|k)$, in a limited control horizon, to the Predictor. Then, the Predictor sub-block predicts the robot state evolution for N_p steps (prediction horizons) through the prediction model (MOD block in Fig. 2), and provides a cost value through the cost function (cf. block in Fig. 2) to the optimizer $\hat{J}(k+i|k)$ in accordance with $\hat{U}(k+i|k)$ and the Predicted Robot State (Rob_{sim}) to the Reference Path Generator in order to recalculate the predicted path. The model uses past information to predict the system response from the future control inputs. These are calculated by the optimizer, which minimizes future errors by optimizing the cost function for ideal control signals. The system constraints are taken into account in the optimization.[16,28]

The iterative minimization process is repeated in a cyclic fashion. Finally, the control output in the first step $U_{best}(k)$ is sent to the robot. To achieve cost function minimization, the NMPC predictor sub-block estimates the evolution of the robot behavior, as well as the global path evolution that is used by the NMPC optimizer and predictor sub-blocks for the cyclic minimization process. After processing the control calculations, the NMPC sends the desired control output back to the robot (controller's reference velocities).

In the early researches on nonholonomic dynamic systems, such as nonholonomic mobile robots, one way found to overcome the computational burden due to the prediction and minimization is the use of models based on neural networks.[34,36,41] To improve precision without increasing the computational cost, it is later tried to parallelize the control with a Proportional, Integrative and Derivative (PID).[37,66] In contrast, the optimization problem is recently treated with a primal-dual neural network.[21,22,54] Finally, in order to model the perturbation in the system, a learning-based NMPC using Gaussian models is developed, also recently.[76,77]

3.1. Linear MPC

Nonholonomic systems are naturally nonlinear. In order to control such a system using a linear controller, one must linearize the model first. Early research works on this subject use prediction models derived from linearized tracking error dynamics to predict future system behaviors.[48] A control law is derived from quadratic cost function involving system tracking error and control effort, including velocity and acceleration constraints to prevent the robot from slipping. When using low-processing embedded systems, a local approach of linear MPC can be used despite the large tracking error.[78] Mobile obstacles avoidance can also be included in the cost function by maximizing the distance between the robot and static obstacles.[101] Also, saturation of the actuators can be considered in the linear MPC approach.[5,106]

Notice that nonhlolonomic systems are not as fast as holonomic systems, which have a higher mobility degree. For these systems, some researches using linear MPC have also been performed.[47,82,104] Other researches focused in controlling robots for outdoor environments using, for example, the Hilare model mobile robot where a high rate of slipping is present[80] or the balancing model where the inertia momentum influences the controller behavior.[8] In nonholonomic systems, the research over the years has focused in robust controllers based on a set of different controller gains for different maneuvers and situations.[12,13] Just recently, approaches based on linear model predictive controllers have also considered uncertainties[31,64] or inherited fast re-planning capability from the D* search algorithm.[89]

3.2. Nonlinear MPC

Since linear (and even successively linearized or time-variant) MPC is not feasible for this mechanical benchmark problem (as linearizing around any fixed point is not controllable and the assumptions for guaranteed stability do not hold), the potential of NMPC techniques is investigated over the years.[26,68] NMPC implementations demonstrate that the cost and reliability of nonconvex optimization are critical issues, making the tuning of controller parameters to be decisive. In general, terminal state constraints are required to guarantee asymptotic stability.[81]

To overcome the on-line optimization issue and to ensure asymptotic convergence of the tracking error, Hedjar *et al.*[40] use a Taylor series approximation in the prediction model. As computational power increases over the years, several researches perform experiments with more complex robot models such as a car-trailer.[9,10,32] Recently, obstacle avoidance has also been a topic of research in NMPC with nonholonomic robots,[11] or omnidirectional robots.[46,73,96]

Finally, in recent researches, stabilization of nonholonomic systems has been studied.[69] Furthermore, the paper by Abbas *et al.*[2] focus on the performance of those controllers with respect to the look-ahead horizon of NMPC in which two different methods of obstacle avoidance are compared and then NMPC is tested in several simulated but realistic tracking scenarios involving static obstacles on constrained roadways.

3.3. NMPC example with differential drive mobile robot

As a proof of concept, we designed an example to demonstrate the efficiency of a NMPC controller applied to the trajectory tracking of a differential mobile robot (Fig. 3). The prediction model used by

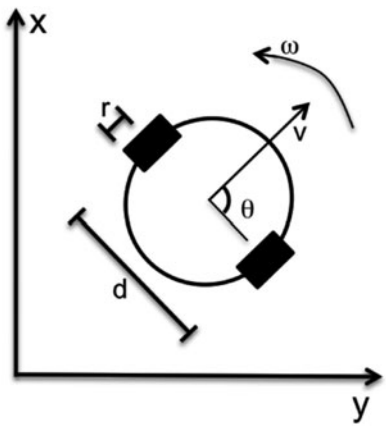


Fig. 3. Typical representation of a two-wheel mobile robot.

the NMPC is the one presented below. In this example, we also use the conjugate gradient optimization algorithm proposed by Conceição *et al.*[19]

According to Fig. 3, the relevant variables for the kinematic model of a typical two-wheel differential mobile robot are its center position coordinates (x_r, y_r), its angle of orientation θ_r, along with its linear and angular velocities (v_r, ω_r), respectively. The classic prediction model derived from the diagram presented in Fig. 3 is described in Eq. 1. The constraint shown in Eq. 2 must be respected in order to avoid lateral spilling from the wheels by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} cos\theta & 0 \\ sin\theta & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v \\ \omega \end{bmatrix}, \tag{1}$$

$$\dot{x}sin\theta - \dot{y}cos\theta = 0. \tag{2}$$

Finally, we can perceive the error states (x_e, y_e, θ_e) used to predict the robot's movement error, which are selected in a rotated coordinate frame and presented in,[39] as follows:

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{ref} - x \\ y_{ref} - y \\ \theta_{ref} - \theta \end{bmatrix}, \tag{3}$$

where a desired reference trajectory is defined by a reference state vector $X_{ref} = [x_{ref}\ y_{ref}\ \theta_{ref}]^T$.

Now, we need to make explicit the relation between linear and angular velocities (v and ω) with the wheel motion. The relation between the angular velocity of each wheel and the robot's linear and angular velocities is shown in Eq. 4, where ω_1 and ω_2 are the angular velocities of the right and left wheels, respectively, r is the wheel radius and d is the length of the robot base. Therefore,

$$\begin{aligned} v &= \frac{1}{2}[r(\omega_1 + \omega_2)], \\ \omega &= \frac{1}{d}[r(\omega_1 - \omega_2)]. \end{aligned} \tag{4}$$

By combining Eqs. 1 and 4, we can obtain a model that expresses the robot's coordinates and orientation in terms of the angular velocity of its wheels, as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{rcos\theta}{2} & \frac{rcos\theta}{2} \\ \frac{rsin\theta}{2} & \frac{rsin\theta}{2} \\ \frac{r}{d} & \frac{-r}{d} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}. \tag{5}$$

Also, Eq. 4 can be easily manipulated to show the direct relation between the control output (the linear and angular velocities corresponding to the desired trajectory) and the input (the right and left wheels' angular velocities). This is shown in Eq. 6:

$$\begin{aligned} \omega_1 &= \frac{1}{r}\left(v + \frac{\omega d}{2}\right), \\ \omega_2 &= \frac{1}{r}\left(v - \frac{\omega d}{2}\right). \end{aligned} \tag{6}$$

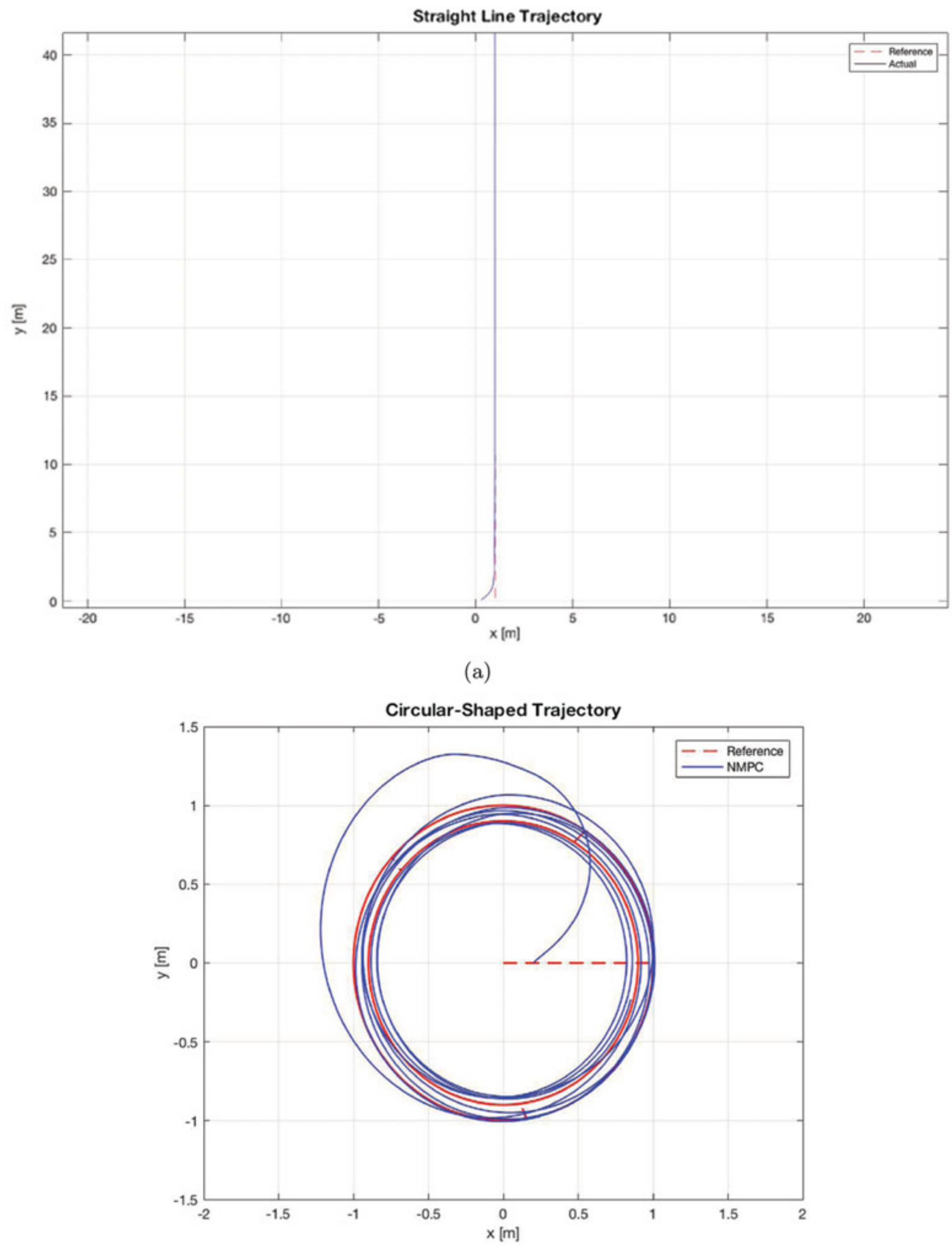
The cost to be minimized by the predictive controller is typically associated to the dynamical change of the system over time. In the trajectory tracking problem, the cost function must penalize the difference between the pose of the robot and the pose of the reference given by the trajectory generator. Additionally, the cost function has a term that penalizes the control effort. Each of these penalizations is associated to a weight that defines the proportion of penalization over the global value of the cost function. The cost function that has to be minimized to control the trajectory of the nonholonomic mobile robot is in general given by

$$\begin{aligned} J(N_1, N_p, N_c) = &\sum_{i=N_1}^{N_p} \lambda_1 \times ([x_{rob}(k+i) - x_{traj}(k+i)]^2 \\ &+ [y_{rob}(k+i) - y_{traj}(k+i)]^2) \\ &+ \sum_{i=N_1}^{N_p} \lambda_2 \times (\theta_{rob}(k+i) - \theta_{traj}(k+i))^2 \\ &+ \sum_{i=1}^{N_c} \lambda_3 \times (\Delta U(k+i-1))^2, \end{aligned} \tag{7}$$

where N_1, N_p are the prediction horizon limits in discrete time, and N_c is the control horizon. λ_1, λ_2, and λ_3 are the weights.

Two trajectories are considered in this example: a straight trajectory and a circular-shaped one. The Pioneer 3 simulated model[65] considers white noise in the values of position and velocities that are fed back to the controllers in order to have a more realistic simulation. The values of λ are $\lambda_1 = 5$, $\lambda_2 = 3$, and $\lambda_3 = 1$ for the straight trajectory and $\lambda_1 = 1$, $\lambda_2 = 1.5$, and $\lambda_3 = 1$ for the circular-shaped trajectory and they were obtained through the method proposed by Nascimento *et al.*[74] The robot model used in the simulation is that of a Pioneer 3 robot. Fig. 4 illustrates the robustness of NMPC even in the presence of noise that is usually treated with filtering approaches. Notice that in the classical NMPC[34] the values for the gains $[\lambda_1, \lambda_2, \lambda_3]$ do not guarantee an optimal solution (J^*).

This simulation just illustrates the application of a NMPC controller in a differential-driven mobile robot subject to nonholonomicity and noise. There are still some improvements to work on as seen in the state-of-the-art works presented in this paper, and we are currently working in some of them, seen next.



(a)

(b)

Fig. 4. Comparative results of NMPC regarding the trajectory tracking problem. (a) Straight trajectory. (b) Circular-shaped trajectory.

4. Variants of Model Predictive Control Applied to Trajectory Tracking Problem of Mobile Robots

The linear MPC is characterized by a family of predictive control strategies in which the dynamic model used for predicting the evolution of the system is linear, even though the dynamics of the closed-loop system can be nonlinear due to the existence of constraints. This simplification is made by considering that the system operates in a controlled area where its behavior is well represented by linear models. However, there are many systems that are inherently nonlinear, in which case a nonlinear model has to be necessarily used. This fact led to the development and use of the NMPC.

In NMPC, the choice of the appropriate model is not the only important issue. Changing to a nonlinear model transforms the control problem from a convex quadratic program to a nonconvex nonlinear problem, which is a much more difficult to solve problem. Furthermore, in the discussed situation there is no guarantee that the global optimum can be found, especially in real-time control when the optimum has to be obtained in prescribed time.[16]

Finally, NMPC has the following disadvantages:

- It is difficult to theoretically prove closed-loop stability;
- It entails large computational costs in the system processing;
- In NMPC, realistic and accurate model of the system is required in order to guarantee accurate predictions of the system behavior.

The current research involving MPC approaches applied to nonholonomic mobile robots can be summarized in four problems: the closed-loop stability, the constraints imposed by sensors (i.e., vision systems), the optimization problem, and the modeling of the mobile robot system. Formation control problems using three robots or more are also a field of study[71,72] but we do not discuss these kinds of problems here. Issues such as obstacle avoidance and robot communication are also still studied but with less effort due to presented solutions along the past years.[73]

4.1. Stability

MPC theory is largely studied nowadays and used in a diverse field of research.[16] But for each type of system, there are different approaches due to systems constraints, and mobile robotics is an example. In mobile robotics, constraints as robot mobility and velocity, unstructured and dynamic environments, model precision, and uncertainties are of such importance that classical control theory does not suffice to solve the path tracking problem of a mobile robot.

In contrast, MPC theory can handle such issues in a robust fashion. Nevertheless, one should take into account the closed-loop stability. By now, linear MPC theory is quite mature[28] and important issues such as stability are well addressed since last decade.[67] Nevertheless, some systems are, in general, inherently nonlinear. Therefore, specially in highly dynamic systems such as mobile robotics, linear models are often inadequate to describe the process dynamics and nonlinear models have to be used. This motivates the use of NMPC. The work by Fontes[29] in 2001 proposes a new MPC framework to generate feedback controls for time-varying nonlinear systems with input constraints. He provided a set of conditions in the design parameters that allows to verify, *a priori*, the stabilizing properties of the considered control strategies. The derived sufficient conditions for stability can also be used to analyze the stability of previous MPC schemes. The addressed class of nonlinear systems is significantly enlarged by removing the traditional assumptions on the continuity of the optimal controls and on the stabilization of the linearized system. Some important classes of nonlinear systems, including some nonholonomic systems, could then be stabilized by MPC.

Thereafter, Gu and Hu[38] introduce a receding horizon (RH) controller used for regulating a nonholonomic mobile robot. The RH control stability is guaranteed by adding a terminal-state penalty to the cost function and a terminal-state region to optimization constraints. A suboptimal solution to the optimization problem is sufficient to achieve stability. In their subsequent work, the same authors[39] develop a RH controller (also called Model Predictive Controller) for tracking control of a nonholonomic mobile robot. The control stability is guaranteed by adding a terminal-state penalty to the cost function and constraining the terminal state to a terminal-state region. The stability analysis in the terminal-state region is investigated, and a virtual controller found. The analysis of the presented results shows that the RH tracking control has simultaneous tracking and regulation capabilities.

Another work that addresses the stability problem in MPC is presented by Yang *et al.*[99] In their work, the authors consider the problem of formation control and obstacle avoidance for a group of nonholonomic mobile robots. On the basis of suboptimal MPC, two control algorithms are proposed. Both algorithms are formulated such that they solve the optimal control problem in which the cost functions are coupled with the dynamics of each interacting robot. A potential function is used to define the terminal-state penalty term, and a corresponding terminal-state region is added to the optimization constraints. Moreover, the main issues, including stability and safety, are also discussed using the Brockett stability theory.[23]

An approach based on Linear Matrix Inequalities (LMI) is proposed by Araújo *et al.*[7] in 2011. The authors present a methodology for state feedback MPC synthesis applied to the trajectory tracking control problem of a three-wheeled omnidirectional mobile robot. The MPC design is based on a cost function developed over finite horizon and LMI framework. The stability of the closed loop system is guaranteed by LMI conditions related with the cost function monotonicity. In contrast, the work of Worthmann *et al.*[98] propose a MPC scheme based on tailored nonquadratic stage cost to solve the problem of steering a nonholonomic mobile robot to a desired position and orientation. They rigorously prove asymptotic stability, while neither stabilizing constraints nor costs are used. To this end, the authors first design suitable maneuvers to construct bounds on the value function. Second, these bounds are exploited to determine a prediction horizon length such that the asymptotic stability of the closed-loop MPC is guaranteed.

In the nonlinear case, the analysis of the terminal cost in the cost function of a NMPC is well studied to analyze its influence over the stability of the NMPC. Several authors propose such modifications in the terminal region.[60,62,81,102] In these works, the stability is achieved but only in simulation, no real robot experiment is performed.

4.2. Vision systems and sensor issues

Another recent issue regarding MPC theory applied to mobile robotics is the sensory system of the robot. Usually, sensors have noise related problems increasing the perception uncertainties. The robot vision system is the one that presents highest perception uncertainties. Used in localization and tracking, a vision system directly influences the controller. This influence causes uncertainties in the robot's position. Usually, this problem is addressed separately in a localization algorithm. Nevertheless, some MPC/NMPC approaches help minimizing these uncertainties.[4] One of the first approaches is the work of Allibert *et al.*[6] that deals with the design of a real-time controller for trajectory tracking in the image plane. The image-based visual servoing (IBVS) task is addressed by a visual predictive approach. The trajectory tracking is formulated as a nonlinear optimization problem in the image plane. The unavoidable constraints in the experiments are taken into account in the design of the predictive control law. A global model, combining the mobile robot and the camera model, is used to predict the behavior of the process.

Regarding model predictive controllers, the vision issue was researched again only in 2013 when Maniatopoulos *et al.*[63] considered the problem of navigating a differential-driven nonholonomic vehicle while maintaining visibility with a stationary target by means of MPC. The approach combines the convergence properties of a dipolar vector field within a constrained NMPC formulation, in which visibility and input saturation constraints are encoded *via* re-centered barrier functions.

Visual servo stabilization of nonholonomic mobile robots has gained extensive attention in recent years. However, the solution of this problem does not consider both the visibility constraints and the actuator limitations, jeopardizing the performance of the controller in practical applications. In their paper, Cao *et al.*[17] introduce a predictive controller for the visual servo stabilization of a mobile robot. The authors propose a kinematic predictive stabilization controller that is used to generate the command of velocity and then they design a dynamic predictive controller in order to make the actual velocity of the mobile robot asymptotically approach the desired one.

Recently, Li *et al.*[55] proposed a visual servo-based MPC method to steer a WMR moving in a polar coordinate toward a desired target. Although visual servo control is not a new field of research, the proposed control scheme introduces the problem using MPC realizing at both kinematics and dynamics levels. The kinematics predictive steering controller generated command of desired velocities that were achieved by employing a low-level motion controller, while the dynamics predictive controller directly generated torques used to steer the WMR to the target.

4.3. Optimization problem

Regarding the controller optimization problem, the above differential-driven mobile robot predictive control approaches use either sub-optimal[38] or genetic algorithm[34,85] solutions, until 2006. Other works focus on theoretical analysis.[1] A good review of efficient solutions of dynamic optimization applied to NMPC problems can be found in the work of Biegler.[15] Furthermore, some of the issues pointed out in this survey such as dealing with uncertainties are still open issues to be investigated when using NMPC specially in mobile robotics.[72]

The first time an optimization problem is addressed in an MPC controller problem is through the work of Lages and Alves.[52] In their work, they propose an optimal control strategy for a differential-drive mobile robot. They state that such a system can not be feedback stabilized by a smooth time-invariant control law. By using MPC, an appropriate control law is implicitly obtained, the system physical constraints on state and inputs are dealt within a straightforward way and the optimization problem is solved by using quadratic programming (QP).

Vougioukas[95] introduces a nonlinear model predictive controller for mobile robots. The basic idea is to use a motion model for the vehicle and to compute, in real-time, an optimal M-step-ahead control sequence, which minimizes the total M+1 step tracking error of the projected motion. In the presence of obstacles, the controller deviates from the reference trajectory by incorporating into the optimization obstacle-distance information from range sensors. The controller's performance strongly depends on parameters such as the optimization horizon M, and the cost-weights assigned to the various tracking errors. The optimization horizon regulates a trade-off between timely obstacle avoidance and tracking quality (large M) versus consistently fast convergence (small M). The cost-weights affect tracking quality and also the shape of the path, by regulating trade-offs among position, orientation, and velocity errors.

In 2008, Conceição *et al.*[19] propose a nonlinear model-based predictive controller (NMPC) for trajectory tracking of a four-wheeled omnidirectional mobile robot. Methods of numerical optimization to perform real-time nonlinear minimization of the cost function are used by them. The cost function penalizes the robot's position error, the robot's orientation angle error, and the control effort. In their work, the conjugate gradient Polak–Ribiere algorithm was used demonstrating a better performance than the steepest descent approach. In the same year, Lim *et al.*[59] present a practical approach for a NMPC scheme with collision avoidance, which is implemented in a mobile robot with two differential wheels. The optimal control input is obtained by solving a discrete nonlinear optimization problem over a pre-described prediction horizon based on a gradient descent method using Lagrange Multipliers. Input and state constraints are implemented using a penalty function. The implemented controller minimizes the cost function through on-line optimization, making it possible to avoid obstacles with a natural and flexible trajectory.

Two years later, Pan and Wang[79] introduce a recurrent neural network (RNN) approach to NMPC. By using decomposition, the original optimization associated with NMPC is reformulated as a QD problem with unknown parameters. They employ the RNN and have developed a learning algorithm for solving the formulated problem.

Still on the NMPC case, Teatro *et al.*[93] propose a NMPC algorithm for on-line motion planning and tracking of an omnidirectional autonomous robot. The formalism in their work is based on a Hamiltonian minimization to optimize a control path as evaluated by a cost function. This minimization is constrained by a nonlinear plant model, which confines the solution space to those paths that are physically feasible. The cost function penalizes the tracking error, the control amplitude, and the presence in a potential field cast by moving obstacles and boards.

Although various tracking control methods with stability have been developed for WMRs, it is still difficult to design optimal or near-optimal tracking controller under uncertainties and disturbances. Recently, a paper by Lian *et al.*[56] propose a near-optimal tracking control method WMRs based on receding-horizon dual heuristic programming (RHDHP). In the proposed method, a backstepping kinematic controller is designed to generate desired velocity profiles, and the RH strategy is used to decompose the infinite-horizon optimal control problem into a series of finite-horizon optimal control problems. In each horizon, a closed-loop tracking control policy is successively updated using a class of approximate dynamic programming algorithms called finite-horizon dual heuristic programming (DHP). In contrast, the optimization process regarding NMPC was addressed by Huang *et al.*[42] where a guidance law based on RH control is developed for reference trajectory tracking to reduce the impact of model errors on guidance performance. This control law aimed to reduce the trajectory deviation caused by the entry guidance of a high-speed vehicles entering a rarefied atmosphere with highly uncertainties, such as the planet Mars. At each guidance cycle, the prescribed trajectory as well as the commanded bank angle in finite horizon is obtained by an indirect optimization method based on Pontryagin's minimum principle. Then a set of algebraic and ordinary differential equations with their boundary conditions, called boundary value problem (BVP), are obtained. In this paper, the BVP is transformed into a system of nonlinear algebraic equations by using the differential transformation method to reduce the computational burden caused by differential operation. The system of algebraic equations is solved by a trust region Newton's method.

Finally, Rosolia *et al.*[87] propose a two-stage nonlinear nonconvex control approach for autonomous vehicle driving during highway cruise conditions. The goal of the controller is to track the centerline of

Table I. Optimization methods used.

Authors	Approach	Experiments
Lages and Alves (2006)[52]	Quadratic programming	Yes
Vougioukas (2007)[95]	Gradient descent (GD)	No
Conceição *et al.* (2008)[19]	Conjugate gradient	Yes
Lim *et al.* (2008)[59]	GD with lagrangian multipliers	No
Pan and Wang (2010)[79]	Neural optimizer	No
Teatro *et al.* (2014)[93]	Hamiltonian minimization	Yes
Lian *et al.* (2016)[56]	Near-optimal tracking with finite-horizon dual heuristic programming	No
Huang *et al.* (2016)[42]	Indirect optimization method based on Pontryagin's minimum principle	No
Rosolia *et al.* (2017)[87]	Generalized minimal residual method augmented with a continuation method	Yes

the roadway while avoiding obstacles. An outer-loopNMPC is adopted for generating the collision-free trajectory with the resultant input based on a simplified vehicle model. The optimization is solved through the generalized minimal residual method augmented with a continuation method. Table I summarizes the optimization approaches that have been proposed through the years. Note that only four of the works perform real robot experiments taking into account real-time constraints. The average mobile robot velocity is around 0.4 m/s that means a highly dynamic system.

4.4. Mobile robot systems models

Given the above discussed optimization problems, regarding MPC theory especially in the nonlinear case, we start discussing about the models used by the model predictive controller to predict the robots behavior. MPC has many research fields regarding issues of the approach itself. Such issues on the model used to predict the systems future behaviors can be described as numerical versus analytical approaches and nonlinear versus linearized versus linear prediction models.

4.4.1. Numerical versus analytical approaches. Success of model-based controllers, such as MPC, depends on having reasonably accurate process models. Often a designed experiment is run to generate the data containing sufficient process excitation needed to accurately identify models.[30] This is a common practice among industry processes models that usually are more complex and difficult to model, but are low dynamic systems. In contrast, real-time highly dynamic systems impose a quick response from the control systems. The case of study brought by our paper is an example. Mobile robots usually need a response from the control systems in less then 10 ms. This imposes an issue to MPC controllers limiting the prediction horizon, the control horizon, and the number of interactions.

However, with the evolution on computer systems and processing units architecture, the computational burden is slowing down. Thus, this evolution allows for classical MPC and NMPC approaches, seen before in slower systems, to be applied on mobile robot systems. Historically, analytical models were used in the Predictor sub-block to predict the mobile robot's future behaviors.[18,29,72] This is mainly because the response time in this dynamic system does not allow a real-time identification model. In contrast, models based on computational intelligence techniques such as neural networks[100] are currently being studied. Furthermore, as the derivation mobile robot models are quite straight forward, model systems identification have never been studied in mobile robot trajectory tracking control using MPC.

Finally, there is a new challenge on MPC and NMPC controllers used also in mobile robotics: state estimation.[50] In mobile robotics, state estimation is crucial for estimating the real state of the system based on several noisy measurements.[94] One recent example is the work by Jayasiri *et al.*[43] where the authors discuss about providing optimization-based solutions to the state estimation and tracking control problems in mobile robotics, specially in nonholonomic robotic systems. The work by Jayasiri *et al.*[43] proposes to solve the estimation problem by using moving horizon estimation (MHE) approach while using a NMPC for solving the tracking control problem. Nevertheless, their approach does not consider the state estimation in the prediction model but outside the controller as a tool only to increase accuracy of feedback data.

4.4.2. Nonlinear versus linearized versus linear prediction models. MPC became an attractive feedback strategy, especially for processes with dominant linear behavior. As above mentioned linear MPC refers to a family of MPC schemes in which linear models are used to predict the system dynamics, even though the dynamics of the closed-loop system is nonlinear due to the presence of constraints. Linear MPC approaches have found successful applications, especially in the process industries, and more than 2200 applications in a very wide range from chemicals to aerospace industries can be fond in the literature.[28] Nowadays, linear MPC theory is mature and important issues such as on-line computation, the interplay between modeling/identification and control and system theoretic issues like stability are well addressed. In contrast, many systems are in general inherently nonlinear. This is mostly true in mobile robotic systems.

In most cases, specially in simulations, the kinematic model of the robot used in the predictor is enough to maintain a small tracking error by the MPC/NMPC controller. Nevertheless, in real robot systems this model does not suffice. Dead-zone, saturation, friction, slippery, and other uncertainties that arise from parameter variations or from neglected dynamics and nonholonomicity are examples of nonlinearities that are difficult to model. And, when it can be modeled, it generally increases the computational cost of the predictive controller.[14]

A simple prediction model means less computational cost and less precision in prediction. One way to perform such prediction with high precision and low computational cost is by using neural models such as neural networks. The paper by Gomez-Ortega and Camacho[34] describes a way of implementing a model-based predictive controller (MPC) for mobile robot path tracking using a nonlinear model of mobile robot dynamics and, thus, allows to calculate an accurate prediction of the future trajectories. Constraints on the maximum attainable speeds are considered by the algorithm. Finally, a multilayer perceptron is used to implement the prediction model in the MPC.

Another approach using neural networks is proposed in 2006 by Yoo *et al*.[100] A generalized predictive control (GPC) method based on self-recurrent wavelet neural network (SRWNN) is proposed for stable path tracking of mobile robots. The SRWNN is used as a model identifier for approximating on-line the states of the mobile robot. Since the control inputs, as well as the parameters of the SRWNN identifier are trained by the gradient descent method with the adaptive learning rates (ALRs), the optimal learning rates that are suitable for the time-varying trajectory of the mobile robot can be found very fast. The ALRs for training the parameters of the SRWNN identifier and those for learning the control inputs are derived from the discrete Lyapunov stability theorem, which are thus used to guarantee the convergence of the system.

One year later, Conceição *et al.*[20] propose a nonlinear model based predictive controller (NMPC) for trajectory tracking of a mobile robot. Their approach was the base for the work of Gao *et al.*[33] Conceição *et al.* latter improved their approach by considering friction in the dynamic model for friction compensation.[18] In this last work, the authors focus on the trajectory tracking control problem of a four-wheeled omnidirectional mobile robot. The controller architecture considers both the kinematic and the dynamic control in a cascade structure, where a model predictive controller (MPC) is used to control the robot dynamics. Part of the control effort is used to compensate the friction effects, allowing for the use of efficient linear MPC algorithms under constraints. In the same year, Ferreira and Moreira[27] propose a NMPC architecture for trajectory tracking of an omnidirectional mobile robot. The controller employs a simplified process model to predict the evolution of the state of the robot, which allows the real-time minimization of the cost function using the gradient descent method.

In contrast, Wei *et al.*[97] study the problem of traction control, i.e., how to stabilize a WMR subject to wheel slippage to a desired configuration. The modes of operation are controlled, whereas wheel slippage, e.g., due to ice, is an autonomous mode change. The WMR can be thus modeled as a hybrid system with both controlled and autonomous switches. MPC for such systems, although robust, typically results in numerical methods of combinatorial complexity. They demonstrate that recently developed embedding techniques can be used to formulate numerical algorithms for the hybrid MPC problem that has the same complexity as MPC for smooth systems.

Finally, Barreto *et al.*[88] introduce and discuss the implementation results of a MPC scheme with friction compensation applied to trajectory tracking of an omnidirectional three-wheeled robot. A cascade structure is used with an inverse kinematics block to generate the velocity references given to the predictive controller. Part of the control effort is used to compensate for the effects of static friction, allowing the use of efficient algorithms for linear MPC with constraints. The last known paper to investigate the slip problem in nonholonomic mobile robots was the work performed by Zhou *et al.*[105] where they took the actual tire side-slip characteristics into account.

A MPC scheme is also developed in the work *via* feedback linearizing method of Li *et al.*[53] to handle the tracking control problem with input saturation. A feedback linearized control law is designed to linearize the nonlinear model, which allows the application of a discrete time linear model predictive control algorithm. Recently, works involving the prediction model with soft constraints,[70] including lateral dynamics,[24] including a extended state observer[103] were investigated. All of these recent researches were in the linear MPC case.

4.4.3. Comparison of prediction models. As seen above, the prediction model of a linear/linearized/nonlinear model predictive controller duels between the balance of low computational cost versus high precision on prediction. Many solutions have arisen over the last years and we summarize them in Table II. Four different main approaches can be used to develop prediction models: linear, linearized, nonlinear, and models based on neural networks. In the nonlinear case, however, there are not many works in the area. The paper by Sigaud *et al.*[92] provides a survey of the corresponding literature with a focus in the on-line regression algorithms for learning nonlinear mechanical models of robots. Recently, Rocha *et al.*[86] applied a Gaussian process model within the MPC to regulate the speed of a heavy-duty truck. This controller takes the variance produced by the Gaussian process model into account in the optimization of the control signal. The proposed controller achieves low tracking error even when working under hard conditions, like steep roads. Gaussian process models are not new inMPC theory,[49] but until the work of Rocha *et al.* they had been applied only to low dynamic systems.[35,57,61]

5. Conclusion and Future Works

MPC theory applied to mobile robotics systems is a very active field. In this paper, we surveyed in particular linear and nonlinear model predictive controllers currently used to solve nonholonomic wheeled mobile robots trajectory tracking problems, providing a unifying view of these approaches. Among other things, we have shown that, despite its popularity and efficiency for some problems, such as controlling the robot's movement along a pre-defined trajectory, MPC/NMPC approaches suffer from several important drawbacks that justify the effort dedicated to look for better algorithms and improvements.

State-of-the-art works presented issues such as control stability, vision, and sensor systems that introduced uncertainties into the control loop, new optimization techniques that could increase the control horizon by minimizing the cost function more rapidly or that guaranties global convergence, and mobile robots prediction model that can be either analytical or numerical approaches and neural based, linear, linearized, or nonlinear models. Despite not being the focus of this work, formation control using MPC as well as MPC applied to other robot configurations such as UUV and Drones can also be found in the literature.

We notice from our studies that there is still room for large performance improvements in this domain just by combining some of the features of these algorithms as well as to capture ideas from the system identification area. From our conclusions, we point out that the application of such researches shall help addressing the "new frontier" of MPC/NMPC in the near future.

Finally, we can state that the most promising subjects for research in nonholonomic wheeled mobile robotics using MPC/NMPC are as follows: the use of the control approach in outdoor roots,

Table II. Predictor model methods comparison.

Authors	Approach	Model type	Experiments
Gomez-Ortega and Camacho (1994)[34]	Neural networks (NN)	Neural	Yes
Yoo *et al.* (2006)[100]	Self-recurrent Wavelet NN	Neural	No
Fontes (2001)[29]	Kinematic and dynamic models	Linear	No
Panathula *et al.* (2012)[80]	Hilare model	Linear	No
Bascetta *et al.* (2016)[14]	Model based on linear fractal transformation	Linear	No
Meiling *et al.* (2016)[70]	Model with soft constraints	Linear	No
Elbanhawi *et al.* (2016)[24]	Model including lateral dynamics	Linear	No
Zhou *et al.* (2016)[105]	Took the actual tire side-slip characteristics into account	Linear	No
Yue *et al.* (2017)[103]	Extended state observer is introduced to evaluate the velocity vectors and model dynamics	Linear	No
Lages and Alves (2006)[52]	Linearized model with QP	Linearized	Yes
Gao *et al.* (2008)[33]	Pure kinematic model	Linearized	No
Barreto *et al.* (2014)[88]	Dynamic model with friction	Linearized	Yes
Li *et al.* (2014)[53]	Model with saturation on actuators	Linearized	No
Conceição *et al.* (2007)[2018]	Dynamic model with friction	Nonlinear	Yes
Ferreira and Moreira (2010)[27]	Simplified model	Nonlinear	No
Wei *et al.* (2013)[97]	Hybrid model with slip	Nonlinear	No

state estimation, minimization of uncertainties from vision and sensor systems (perception driven), and control stability.

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